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A Wave Approach to the Noise Properties of Linear Microwave Devices

R. P. MEYS

Abstract—**Noise temperature or noise factor are important parameters for many microwave devices. Their dependence on source characteristics is classically established using low-frequency concepts such as impedance, admittance, voltage, and current sources. This paper presents a derivation of the noise properties of linear two-ports in terms of noise waves, which leads to a convenient measurement method in distributed systems.**

I. INTRODUCTION

THE classical derivations for noise temperature and noise factor of linear two-ports use the equivalent circuit of Fig. 1(a) and lead to the results [1], [2]

$$T_n = T_{n\,\min} + T_0 \frac{R_n}{G_s} \, |Y_s - Y_{\rm opt}|^2 \tag{1}$$

$$F_0 = F_{0\,\min} + \frac{R_n}{G_s} \, |Y_s - Y_{\rm opt}|^2. \tag{2}$$

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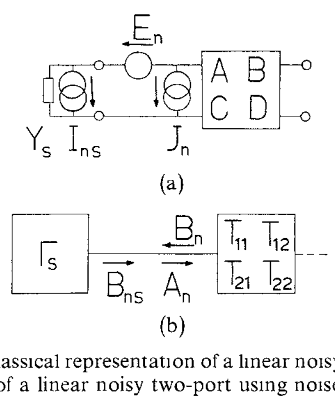

Fig. 1. (a) The classical representation of a linear noisy two-port. (b) The representation of a linear noisy two-port using noise waves.

Sometimes (1) and (2) are written as

$$T_n = T_{n\,\min} + 4T_0 \frac{R_n}{Z_0} \frac{|\Gamma_s - \Gamma_{\rm opt}|^2}{|1 + \Gamma_{\rm opt}|^2 (1 - |\Gamma_s|^2)} \tag{3}$$

$$F_0 = F_{0\,\min} + 4 \frac{R_n}{Z_0} \frac{|\Gamma_s - \Gamma_{\rm opt}|^2}{|1 + \Gamma_{\rm opt}|^2 (1 - |\Gamma_s|^2)} \tag{4}$$

with

$\Gamma_{\text{opt}} = (Y_0 - Y_{\text{opt}})/(Y_0 + Y_{\text{opt}})$;
$Z_0 = (1/Y_0) =$ characteristic impedance of the distributed system.

These last expressions are hybrid ones, containing the impedance parameter R_n and the wave parameter Γ_{opt}. The unknown quantities $T_{n\,\text{min}}$ ($F_{0\,\text{min}}$), R_n, Y_{opt} (Γ_{opt}) are determined either a) by a cut-and-try method using an automatic noise-figure meter, or b) by a set of measurements with various source admittances followed by a computation such as described in [2].

At microwave frequencies, however, a treatment of noise in terms of waves looks attractive. This was already done [3]. The analysis starts with the model of Fig. 1(b) and defines two uncorrelated noise waves A_n, B_n by

$$A_n = -\frac{E_n + Z_\nu J_n}{2\sqrt{\text{Re}\,(Z_\nu)}}$$

$$B_n = \frac{E_n - Z_\nu^* J_n}{2\sqrt{\text{Re}\,(Z_\nu)}}$$

where Z_ν is a complex normalization impedance dependent on the device. This choice leads to theoretical simplicity, but does not suggest an easy-to-use measurement method.

II. Noise Representation by Correlated Wave Sources

Let us start with the same model [Fig. 1(b)], but simply define the noise-wave sources with reference to the characteristic impedance Z_0 of the line or waveguide mode used at the input. We now connect a source of reflection factor Γ_s and noise wave B_{ns} to the input. The total noise wave that would be incident on a noiseless matched load substituted for the device input is

$$A_{ns} = A_n + \Gamma_s B_n + B_{ns}$$

the squared modulus of which equals, assuming no correlation between source and two-port noise

$$\overline{|A_{ns}|^2} = \overline{|A_n|^2} + |\Gamma_s|^2 \overline{|B_n|^2} + 2\,\text{Re}\,(\Gamma_s \overline{A_n^* B_n}) + \overline{|B_{ns}|^2}. \quad (5)$$

If we introduce the following notation:

$$\overline{|A_{ns}|^2} = kT_{ns}\Delta f$$

$$\overline{|A_n|^2} = kT_a\Delta f$$

$$\overline{|B_n|^2} = kT_b\Delta f$$

$$\overline{A_n^* B_n} = kT_c\Delta f \cdot e^{j\phi_c}$$

$$\Gamma_s = |\Gamma_s| e^{j\phi_s}$$

where k is Boltzmann's constant and Δf is the frequency interval of interest, and consider that the source noise wave is given by

$$\overline{|B_{ns}|^2} = (1 - |\Gamma_s|^2)kT_s\Delta f$$

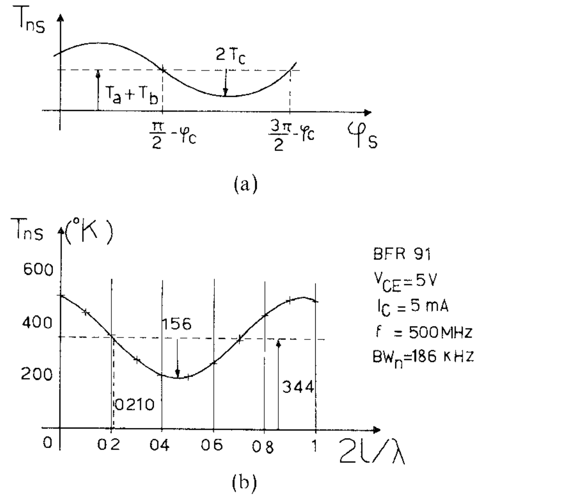

Fig 2 (a) The total direct noise wave as a function of ϕ_s (b) An example of experimental results

(5) becomes

$$T_{ns} = T_a + |\Gamma_s|^2 T_b + 2T_c|\Gamma_s|\cos(\phi_s + \phi_c) + T_s(1 - |\Gamma_s|^2). \quad (6)$$

To entirely define the noise properties of the two-port, we must determine the four quantities which appear in (6): T_a, T_b, T_c, ϕ_c. The following procedure is proposed.

a) Connect an almost lossless source ($|\Gamma_s| \simeq 1$) of variable ϕ_s to the input. Then observe

$$T_{ns} = T_a + T_b + 2T_c \cos(\phi_s + \phi_c)$$

which is represented as a function of ϕ_s in Fig. 2(a). From this graph we deduce T_c, ϕ_c, $T_a + T_b$.

b) Connect a matched load of known temperature T_s to the input, so

$$T_{ns} = T_a + T_s$$

from which we deduce T_a.

III. Example of Noise-Wave Sources Measurement

The method has been applied as an example to a BFR91 transistor operating at $V_{CE} = 5$ V, $I_c = 5$ mA to determine its noise-wave sources around $f = 500$ MHz. The test setup for the measurement of T_{ns} with $|\Gamma_s| \simeq 1$ is shown in Fig. 3(a). A waveguide below the cutoff attenuator is used in the reversed mode to inject a known sine wave. As the coupling is small and a short is connected at the third port, we realize very closely the condition $|\Gamma_s| = 1$. A variable-length line is placed between the attenuator and the DUT. The receiving system consists of a 500-MHz selective preamplifier, a first mixer and 50-MHz IF amplifier, a step attenuator, a second mixer and 5-MHz IF amplifier with calibrated noise bandwidth, and finally a quadratical diode detector. First, with the IF attenuator on its reference position, the detector output is recorded. Then the IF attenuation is increased by A_{d} and a known sine wave is injected so as to recover the

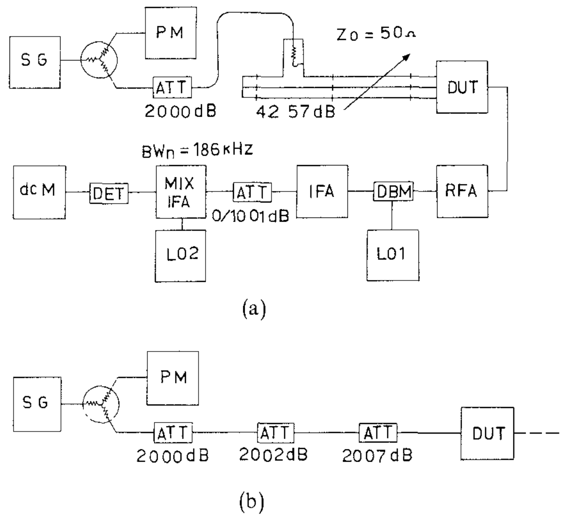

Fig. 3. (a) The measurement setup with $|\Gamma_s| = 1$ (b) The measurement setup with $|\Gamma_s| = 0$.

TABLE I
THE NUMERICAL RESULTS PERTAINING TO THE EXAMPLE

l(mm)	0	30	60	90	120	150	180	210	240	270	300
P_m(-dBm)	46 80	47 34	48 28	49 59	50 70	50.80	49.85	48.49	47.78	46 88	46.86
T_{ns}(°K)	499	441	355	263	203	199	247	338	427	490	492

same detector output as before. It is shown in the Appendix that under these conditions

$$T_{ns} = \frac{1}{A_{\rm if}^2 - 1} \frac{1}{A_s^2} \frac{P_m}{kBW_n} \tag{7}$$

$A_{\rm if}^2$ IF power attenuation;
A_s^2 total power attenuation between power meter and DUT input;
BW_n system-noise bandwidth;
P_m power-meter reading.

The experimental results are shown in Table I. Fig. 2(b) is a plot of T_{ns}, as calculated from (7) with the attenuation and bandwidth values indicated on Fig. 3(a), versus the normalized variable-line length $2l/\lambda$. Conformity of the experimental curve to the theoretical sinusoid is almost perfect. From Fig. 2(b) we find

$$T_a + T_b = 344 \text{ K}$$

$$T_c = 78 \text{ K}$$

$$\frac{3\pi/2 - \phi_c}{2\pi} = 0.703 - 0.210$$

where the 0.703 term in the last equation is the phase of Γ_s for zero extension, which has been measured separately. The minus sign follows from the fact that arg (Γ_s) decreases as the line is extended.

The last step is to measure T_a. This step is, in fact, similar to the conventional measurement of noise temperature with a matched source. It has been performed by modifying the test setup as shown in Fig. 3(b). Using the relation

$$T_a = \frac{1}{A_{\rm if}^2 - 1} \frac{1}{A_s^2} \frac{P_m}{kBW_n} - T_s \tag{8}$$

(which is a special case of (7)), with T_s equal to the ambient temperature (301 K), $P_m = -49.51$ dBm, and with the attenuation values indicated on Fig. 3(a) and (b), we find $T_a = 172$ K.

IV. FROM NOISE WAVES TO NOISE TEMPERATURE AND NOISE FACTOR

The noise temperature T_n is defined as that temperature which must be added to the source temperature to account for the noise introduced by the linear two-port. Using this definition and (6) we obtain

$$T_n = \frac{T_a + |\Gamma_s|^2 T_b + 2T_c |\Gamma_s| \cos(\phi_s + \phi_c)}{1 - |\Gamma_s|^2}. \tag{9}$$

Let us now write (3) under the more compact form

$$T_n = T_{n\,\min} + T_d \frac{|\Gamma_s - \Gamma_{\rm opt}|^2}{1 - |\Gamma_s|^2}. \tag{10}$$

Identification of (9) and (10) yields the results

$$T_a = T_{n\,\min} + T_d |\Gamma_{\rm opt}|^2 \tag{11}$$

$$T_b = T_d - T_{n\,\min} \tag{12}$$

$$T_c = T_d |\Gamma_{\rm opt}| \tag{13}$$

$$\phi_c = \pi - \phi_0 \tag{14}$$

with $\phi_0 = \arg(\Gamma_{\rm opt})$. Inversion of the system of (11)–(14) yields

$$T_d = \tfrac{1}{2}\left(T_{ab} \pm \sqrt{T_{ab}^2 - 4T_c^2}\right), \qquad (T_{ab} = T_a + T_b) \tag{15}$$

$$T_{n\,\min} = T_d - T_b \tag{16}$$

$$|\Gamma_{\rm opt}| = \frac{T_c}{T_d} \tag{17}$$

$$\phi_0 = \pi - \phi_c. \tag{18}$$

It is a simple matter to verify that only the + sign in (15) must be retained to satisfy the condition $|\Gamma_{\rm opt}| < 1$. The corresponding equations for noise factor are readily obtained, if we bear in mind that

$$F_0 = 1 + \frac{T_n}{T_0}.$$

When applying (15)–(18) to our previous example, we find successively

$$T_d = 325 \text{ K}$$

$$T_{n\,\min} = 153 \text{ K}$$

$$|\Gamma_{\rm opt}| = 0.240$$

$$\phi_0 = 87.5^\circ$$

which achieves the noise characterization of the device.

V. Conclusion

A description of the noise behavior of linear microwave devices has been proposed based on the four fundamental wave parameters T_a, T_b, T_c, ϕ_c. As the example has shown, they can be easily measured and lead, through simple formulas, to the traditional quantities T_n and F_0. Sine waves have been used as power reference, but this is in no case a necessity. Use of calibrated noise sources is possible insofar as a sufficient power level is available to inject a noise wave of the same order of magnitude as the device's sources despite the small coupling required by the condition $|\Gamma_s| \simeq 1$. This usually avoids the need for working with a known noise bandwidth.

Appendix

If Γ_i designates the reflection factor at the device's input, the total incident noise wave actually is

$$\frac{1}{1-\Gamma_s\Gamma_i} A_{ns}$$

giving rise at the output of the measurement setup to a power

$$P_0 = \alpha \frac{1}{|1-\Gamma_s\Gamma_i|^2} \overline{|A_{ns}|^2}. \tag{19}$$

When a sine wave B_{ss} is injected, the total incident wave becomes

$$\frac{1}{1-\Gamma_s\Gamma_i}(A_{ns}+B_{ss}).$$

Following the principle of the measurement, this wave will give rise to the same output power as before, after attenuation by A_{if}, so

$$P_0 = \alpha \frac{1}{A_{\mathrm{if}}^2} \frac{1}{|1-\Gamma_s\Gamma_i|^2} \left(\overline{|A_{ns}|^2} + |B_{ss}|^2\right). \tag{20}$$

From (19) and (20) it follows that

$$\overline{|A_{ns}|^2} = \frac{1}{A_{\mathrm{if}}^2 - 1} |B_{ss}|^2. \tag{21}$$

Substituting

$$|B_{ss}|^2 = \frac{P_m}{A_s^2} \quad \text{and} \quad \overline{|A_{ns}|^2} = kT_{ns}BW_n$$

in (21) leads to (7).

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